

Dark Matter and Inflation in $R + \zeta R^2$ Supergravity

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As is well known, the gravitational degrees of freedom contained in $R + \zeta R^2$ (super)gravity lead to Starobinsky's potential, in a one-field setting for inflationary Cosmology that appears favored by Planck data. In this letter we discuss another interesting aspect of this model, related to gravitino production, with emphasis on the corresponding mass spectrum. Assuming that supersymmetry is broken at a very high scale, Super Heavy Gravitino Dark Matter (SHGDM) and Starobinsky's inflation can be coherently unified in a $R + \zeta R^2$ supergravity. Gravitinos are assumed to be the Lightest Supersymmetric Particles (LSP) and are non-thermally produced during inflation, in turn originated by a scalar with a Starobinsky's potential. Gravitino mass runs with the inflaton field, so that a continuous spectrum of superheavy gravitinos emerges. The theory is implemented with a $U(1)_R$ gauge symmetry. However, in a string UV completion, $U(1)_R$ -symmetry can be broken by non-perturbative string instantons, while for consistency of our scenario $U(1)_R$ gauge symmetry breaking must be broken in order to generate a soft mass terms for the gravitino and gauginos. R-parity violating operators can be generated at non-perturbative level. Gravitinos can decay into very energetic neutrinos and photons in cosmological time scale, with intriguing implications for high energy cosmic rays experiments.

I. INTRODUCTION

Starobinsky's model is the simplest $f(R)$ -extension of the Einstein-Hilbert action [1]. As is well known, Starobinsky's $R + \zeta R^2$ is a good theory of inflation, in agreement with recent Planck data [2]. Starobinsky's model can be conformally transformed into a scalar-tensor theory, where the scalaron has an inflation slow-roll potential. This motivated theoretical researches of a supergravity embedding the Starobinsky's model. The simplest proposal suggested in Ref.[3, 4] entails a tachyonic instability of the Goldstino at large values of the inflaton. On the other hand, this problem was solved in Ref.[5, 6] and in Ref.[7–23] in frameworks of no-scale and Volkov-Akulov supersymmetry. As a result, a consistent $R + \zeta R^2$ supergravity can be obtained without unstable moduli fields. This is a necessary (but not sufficient) condition for a UV completion in string theories, where many other scalar moduli of the compactification are inevitably introduced. However, new implications of $R + \zeta R^2$ supergravity for the non-supersymmetric Starobinsky's model were not fully addressed in literature. For instance, possible implications of this model on dark matter production were not been analyzed in all the details.

In this paper we study $R + \zeta R^2$ supergravity with local supersymmetry broken at scales higher than the inflaton reheating. As is well known, a supergravity theory has to

contain at least a new supersymmetric spin 3/2 partner for the graviton, the gravitino. If SUSY is broken at high scales, the gravitino will naturally get a large soft mass term comparable to the SUSY scale. The gauge R-parity symmetry protects the gravitino against R-parity violating couplings. The gravitino could decay in R-preserving transitions into other Supersymmetric Standard Model (or Beyond SM) particles. However, we assume that gravitino is the Lightest Supersymmetric Particle (LSP) of the supersymmetric spectrum. On the other hand, $U(1)_R$ symmetry cannot be preserved at lower energies, otherwise gauginos cannot have soft mass terms larger than the gravitinos mass. We shall comment on possible Stückelberg mechanism for $U(1)_R$ breaking, associated to the presence of non-perturbative effects like exotic D-brane instantons in open strings theories. These aspects were recently discussed in the context of intersecting D-brane models and quiver theories, where exotic instantons can generate $B - L$ violating couplings with intriguing implications for rare process and baryogenesis [24–40]. In this scenario, the gravitino problem is avoided: the gravitino is a supermassive particle which does not decay in a time $t \leq 10\text{ s} \div 20\text{ min}$, so that Big Bang Nucleosynthesis is not affected and ruined [41–43]. We will show how a super-heavy gravitino can be non-thermally produced during Starobinsky's inflation, with the right Cold Dark Matter abundance. On the other hand thermal production of gravitinos will be suppressed if the gravitino is heavier than the inflaton mass [49]. As a result, Cold Dark Matter is connected to the parameter space of inflation in a unified minimal framework.

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Finally, we shall discuss how gravitinos can be destabilized in cosmological time scales, decaying into very high energy neutrinos, which could be detectable in principle by high energy cosmic rays experiments: AUGER, Telescope Array, ANTARES and IceCube.

II. $R + \zeta R^2$ SUPERGRAVITY

Let us consider the Lagrangian of $R + \zeta R^2$ supergravity coupled to matter [14–17, 20]

$$\mathcal{L} = -[-L\mathcal{V}_R + L\Phi(z, \bar{z})]_D + \zeta[\mathcal{W}_\alpha(\mathcal{V}_R)\mathcal{W}^\alpha(\mathcal{V}_R)] \quad (1)$$

$$\mathcal{V}_R = \log \frac{L}{S\bar{S}}$$

where the first term contains the standard Einstein-Hillbert action while the R^2 term is generated by the kinetic term of the real superfield \mathcal{V}_R , S is the compensator field of old minimal supergravity, the functional $\Phi(z, \bar{z})$ is the Kähler potential of the z scalar fields and L is the linear multiplet. Eq.(1) can be rewritten as

$$\zeta\mathcal{W}_\alpha(\mathcal{U})\mathcal{W}^\alpha(\mathcal{U}) - \left[\mathcal{S}_0\bar{\mathcal{S}}_0 e^{\mathcal{U}-\mathcal{T}-\bar{\mathcal{T}}} \left(\mathcal{U} + \frac{1}{3}\Phi - \mathcal{T} - \bar{\mathcal{T}} \right) \right]_{+c.c.} \quad (2)$$

where $\mathcal{S} = \mathcal{S}_0 e^{-\mathcal{T}}$ and $\mathcal{T} + \bar{\mathcal{T}}$ are lagrangian multiplier fields which allow to consider an unconstrained vector multiplet \mathcal{U} . The gauged R-symmetry can be implemented as

$$\mathcal{V}_R \rightarrow \mathcal{V}_R + \Omega + \bar{\Omega}, \quad z_I \rightarrow e^{q_I \Omega} z_I, \quad \mathcal{S} \rightarrow e^{-\Omega} \mathcal{S} \quad (3)$$

with Ω chiral superfield. A superpotential compatible with R-symmetry can be included as a term $\mathcal{S}_0^3 e^{-3\mathcal{T}} \mathcal{W}(z)$ in the lagrangian (2).

The scalar potential is a sum of V_F and V_D :

$$V_F = e^{\mathcal{G}} (\mathcal{G}_A \mathcal{G}^{AB} \mathcal{G}_B - 3), \quad \mathcal{G}_A = \frac{\partial \mathcal{G}}{\partial Z^A}, \quad Z^A = (T, z^I) \quad (4)$$

$$\mathcal{G} = K + \log(e^{-3\mathcal{T}} \mathcal{W}) + \log(e^{-3\bar{\mathcal{T}}} \bar{\mathcal{W}})$$

$$2\zeta V_D = 2\mathcal{G}_T + \sum [q_I z^I \mathcal{G}_I + q_I \bar{z}^I \bar{\mathcal{G}}_I] \quad (5)$$

The old Starobinsky's inflaton potential can be recovered assuming that the F-term is so steep to rapidly drive all z^I fields to $\mathcal{W}_I = \partial \mathcal{W} / \partial z^I \rightarrow 0$, which is an R-symmetric vacuum. The only contribution to the potential comes from the D-term:

$$V = \frac{1}{a} \left(\frac{3}{\mathcal{X}} - 3 \right)^2 = \frac{9}{a} \left[e^{-\sqrt{2/3}\phi} - 1 \right]^2 \quad (6)$$

where

$$\mathcal{X} = e^{\sqrt{2/3}\phi} = \mathcal{T} + \bar{\mathcal{T}} - \frac{1}{3}\Phi \quad (7)$$

which corresponds to the Starobinsky's potential, as mentioned above (all scalar fields are adimensionalized in Planck units).

The off-shell formulation of the minimal Starobinsky lagrangian during inflation is determined by

$$\mathcal{K} = -3 \log[\mathcal{T} + \bar{\mathcal{T}} - \Phi(z, \bar{z})], \quad \mathcal{W}_I \rightarrow 0 \quad (8)$$

The corresponding gravitino mass is

$$m_{\tilde{G}} = e^{\mathcal{K}/2} \frac{\mathcal{W}}{M_{Pl}^2} = e^{-\sqrt{3/2}\phi} \frac{\mathcal{W}}{M_{Pl}^2} \rightarrow 0 \quad (9)$$

We shall now assume that supersymmetry is spontaneously broken at scales higher than the inflation reheating, so that the superpotential \mathcal{W} can be set to a constant $\mathcal{W}_0 > 0$. As a consequence, a continuous spectrum of gravitinos will be produced during the inflation, with an average mass of

$$\langle m_{\tilde{G}} \rangle \simeq \langle e^{-\sqrt{3/2}\phi} \rangle_{\Delta N} \frac{\mathcal{W}_0}{M_{Pl}^2} \simeq 0.15 \frac{\mathcal{W}_0}{M_{Pl}} \quad (10)$$

taking into account that the the inflationary plateau has a width of $\Delta\phi \simeq 5M_{Pl}$ corresponding to $\Delta N = \log a_f/a_i \simeq 60$ e-folds of slow-roll inflation. A useful first approximation is to set $\langle \phi \rangle \simeq \Delta\phi/2$. In particular, $\phi(t_R) \simeq M_{Pl}$ while $\phi(t_R - \Delta t) \simeq 6M_{Pl}$ with a Δt time scale corresponding to ΔN . As a consequence, Eq.(9) implies that a spectrum of massive gravitinos with $m_{\tilde{G}} \simeq 2 \times (0.4 \times 10^{-4} \div 1) \langle m_{\tilde{G}} \rangle$ is generated during slow roll. Fig.1 displays the precise Gravitino mass as a function of the inflaton field.

A. Comments on the vacuum state with spontaneously broken R-symmetry and SUSY

As mentioned above, in Starobinsky's supergravity, the condition $\mathcal{W}_I \rightarrow 0$ during inflation is a viable way-out to the second modulus problem. The superpotential rolls down to zero before the inflation epoch. For $\mathcal{W}_I = 0$, the vacuum state is R-symmetric and SUSY during the inflation stage. This condition avoids any dangerous dynamics of the second modulus field, potentially ruining conditions for a successful inflation. The condition $\mathcal{W}_I = 0$ implies a massless gravitino during inflation, which is incompatible with our suggestion. On the other hand, the spontaneous symmetry breaking of $U_R(1)$ and SUSY, before or at least during inflation epoch while after the fast rolling down of the superpotential, can only generate a constant contribution to the superpotential as $\rightarrow \mathcal{W}_0 = \text{const} \neq 0$. This implies that the \mathcal{G} -term gets an extra contribution

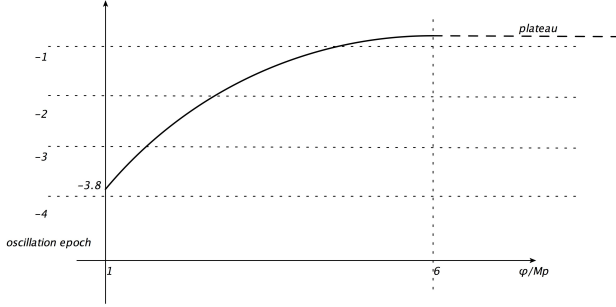
$$\Delta \mathcal{G} = \log \mathcal{W}_0 + \log \bar{\mathcal{W}}_0 = \text{const} \quad (11)$$

which implies a constant shift of the V_F -term as

$$\Delta V_F = -3\mathcal{W}_0 \bar{\mathcal{W}}_0 \quad (12)$$

(only dependent by derivative of \mathcal{G}) and a constant shift of the V_D -term as $2\zeta\Delta V_D = -12$. As a consequence, the inflaton potential is only shifted by a constant factor. These numerical factors are not very important: they can be reabsorbed in the normalization of the Starobinsky's potential, as often discussed in literature. So that, the spontaneous symmetry breaking of $U_R(1)$ and SUSY cannot contribute with dynamical interactions term to \mathcal{W}_I , i.e. it cannot destabilize the second modulus field. For instance, the R-symmetry implemented in Eqs.(3) has fixed the structure of the potential Eq.(6) under the condition on \mathcal{W}_I . One can see that the only effect of a $\mathcal{W}_0 = \text{const} \neq 0$ during the inflation is the shift of the potential Eq.(6) of a constant factor and the z_I fields remain stabilized.

FIG. 1. Gravitino mass function of Starobinsky inflaton. In the x-axis, the inflaton field is conveniently normalized in Planck units, while in the y-axis the gravitino mass function is normalized with respect of the average gravitinos mass $\langle m_{\tilde{G}} \rangle$ (in \log_{10} scale in the y-axis). In particular, the oscillating epoch effectively starts at $\phi/M_P \simeq 1$. On the other hand, the slow-roll effectively starts at $\phi/M_P \simeq 6$. $\Delta\phi/M_P \sim 1 \div 6$ is the gravitino production epoch. So that, a continuous spectrum of super-heavy gravitinos is produced.



III. GRAVITINO AS SHDM

In this section, we shall discuss how a correct Cold Dark Matter (CDM) abundance can be recovered in $R + \zeta R^2$ supergravity. In particular, we shall discuss the non-thermal production of gravitinos during inflation. The non-thermal Super Heavy Dark matter production triggered by inflation was studied in the simpler case of a scalar DM particle in [44]. However, this mechanism can be implemented for gravitinos, even formally more subtle and never discussed in literature by other authors.

First of all, the gravitino field in the full $R + \zeta R^2$ supergravity is described by the Rarita-Schwinger action in presence the of a FRW dynamical metric:

$$S = \int d^4x e \bar{\psi}_\mu \mathcal{R}^\mu[\psi] \quad (13)$$

$$\mathcal{R}^\mu[\psi] = i\gamma^{\mu\nu\rho} \mathcal{D}_\nu \psi_\rho + m_{\tilde{G}} \gamma^{\mu\nu} \psi_\nu \quad (14)$$

$$\mathcal{D}_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{4} \omega_{\mu ab} \gamma^{ab} \psi_\nu - \Gamma_{\mu\nu}^\rho \psi_\rho \quad (15)$$

where $\gamma^{\mu_1 \dots \mu_n} = \gamma^{[\mu_1} \dots \gamma^{\mu_n]}$, $e = \det e_\mu^a$ and e_μ^a is the (inverse) vielbein. We can assume a torsion-free background metric so that $\Gamma_{\mu\nu}^\rho = \Gamma_{\nu\mu}^\rho$.

The EoM is

$$(i\not{D} - m_{\tilde{G}})\psi_\mu - \left(i\mathcal{D}_\mu + \frac{m_{\tilde{G}}}{2}\gamma_\mu\right)\gamma \cdot \psi = 0 \quad (16)$$

In a FRW cosmological background during inflation, Eq.(16) is reduced to

$$i\gamma^{\mu\nu}\partial_\mu \psi_\nu = -\left(m_{\tilde{G}} + i\frac{a'}{a}\gamma^0\right)\gamma^\mu \psi_\mu \quad (17)$$

with

$$e_\mu^a = a(\eta)\delta_\mu^a, \quad m_{\tilde{G}} = m_{\tilde{G}}(\eta), \quad \omega_{\mu ab} = 2\dot{a}a^{-1}e_{\mu[a}e_{b]}^0, \quad (18)$$

and the solution reads

$$\psi_\mu(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k_0} \sum_\lambda \{e^{i\mathbf{k}\cdot\mathbf{x}} b_\mu(\eta, \lambda) a_{k\lambda}(\eta) \quad (19)$$

$$+ e^{-i\mathbf{k}\cdot\mathbf{x}} b_\mu^C(\eta, \lambda) a_{k\lambda}^\dagger(\eta)\}$$

The corresponding mode equation is

$$\hat{P}^\mu b_\mu(\eta, \lambda) = 0 \quad (20)$$

$$\hat{P}^\nu = i\gamma^{\mu\eta}\partial_\eta - \gamma^{\mu i}k_i - \left(m_{\tilde{G}} + i\frac{a'}{a}\gamma^0\right)\gamma^\nu \quad (21)$$

Applying \hat{P}^ν on Eq.(20) as

$$\hat{P}_\nu \hat{P}^\mu b_\mu(\eta, \lambda) = 0 \quad (22)$$

we can rewrite the equation for modes in form

$$b_\mu''(\eta, \lambda) + \omega^2(k, a) b_\mu(\eta, \lambda) = 0 \quad (23)$$

where $b'' \equiv \partial^2 b(\eta, \lambda)/\partial \eta^2$. The Eq.(20) can be rescaled as

$$b_\mu''(\hat{\eta}, \lambda) + \omega^2(\hat{k}, \hat{a}) b_\mu(\hat{\eta}, \lambda) = 0 \quad (24)$$

where $\mu = m_{\tilde{G}}/H_e$, $\hat{\eta}/(a_e H_e) = \eta$, $\hat{a} = a/a_e$ and H_e, a_e correspond to the oscillation epoch quantities - we have chosen this normalization for convenience. The EoM can be solved imposing the boundary conditions. Let us comment that the fixing of boundary conditions corresponds to fixing the vacuum state. In order to calculate the number density of gravitinos produced, we perform the Bogoliubov transformation from the vacuum mode solution with boundary $\eta = \eta_0$ - corresponding to the initial cosmological time at which the vacuum state is specified- into the vacuum mode solution of boundary $\eta = \eta_1$ - corresponding to a generic later time at which

gravitinos are no longer promoted from virtual to real particles (roughly we can assume $\phi \simeq M_{Pl}$ or so, close to the oscillation epoch). Let us note that the exact numerical values of $\eta_{0,1}$ are not important in the dynamical region $a'/a^2 \ll 1$ or $\mu a/k \ll 1$. In this approximation, the EoM will be integrated with $\eta_0 = -\infty$ and $\eta_1 = +\infty$. We can define the Bogoliubov transformation as

$$b_\mu^{(\eta_1)}(\hat{\eta}, \lambda) = \alpha_k b_\mu^{(\eta_0)}(\eta, \lambda) + \beta_k b_\mu^{(\eta_0)^C}(\eta) \quad (25)$$

where $b_\mu^{(\eta_1)}$ is the mode coefficient fixed on the Cauchy surface $\eta = \eta_1$ while $b_\mu^{(\eta_0)}$, $b_\mu^{(\eta_0)^C}$ are fixed on the Cauchy surface $\eta = \eta_0$, where α_k, β_k are the Bogoliubov's coefficient for 4-momenta k . One can estimate the energy density of the gravitinos produced during inflation as (see Appendix of Ref.[44] for a similar estimation)

$$\rho_{\tilde{G}}(\eta_1) = \langle m_{\tilde{G}} \rangle n_{\tilde{G}}(\eta_1) = \langle m_{\tilde{G}} \rangle H_e^3 \left(\frac{1}{a(\eta_1)} \right) \mathcal{P} \quad (26)$$

with

$$\mathcal{P} = \int_0^\infty \frac{dk}{2\pi^2} k^2 |\beta_k|^2 \quad (27)$$

where we performed a Bogoliubov transformation from the Cauchy surface foliated by $\eta = \eta_0$ to another Cauchy surface with cosmological frame time $\eta_1 > \eta_0$ and assuming the inflation conditions $\dot{a}/a^2 \ll 1$. As mentioned above, for formal convenience, we have normalized $k \rightarrow k/aH_e$, $\eta \rightarrow \eta/a_e H_e$, $a \rightarrow a/a_e$, where e labels variables of the oscillation epoch. As usual, in such a procedure there is an apparent ambiguity in the definition of the vacuum. As mentioned above, such a problem is equivalent to the definition of the boundary conditions for Eq.(17). We remind that a systematical method of classification of the inequivalent vacuum states was suggested in Refs.[45–47], introducing the concept of adiabatic vacuum. From such a definition, it is possible to construct a set of solutions for the EoM (17) reduced to the usual plane waves ($a'(\eta) = 0$, for all η values). Let us define the n -th adiabatic vacuum at a certain time η^* by following boundary conditions:

$$b_\mu(\eta^{(n)}) = b_\mu^{(n)}(\eta^*), \quad b'_\mu(\eta^*) = b_\mu'^{(n)}(\eta^*) \quad (28)$$

where $b_\mu^{(n)}(\eta)$ is a n -th order perturbative expansion of the complete solution, satisfying the n -th adiabatic order in the asymptotic limit (see Ref.[48] for a general and more detailed definition).

We estimate the relation among the gravitino energy density normalized over the radiation as:

$$\frac{\rho_{\tilde{G}}(t_0)}{\rho_R(t_0)} = \frac{\rho_{\tilde{G}}(t_{Re})}{\rho_R(t_{Re})} \left(\frac{T_R}{T_e} \right) \quad (29)$$

where $\rho_{\tilde{G}}(t_{Re})/\rho_R(t_{Re})$ is determined after the Reheating epoch, and t_0 is the present cosmological time. Gravitinos were produced during the $t_e > t_{Rh}$ epoch, i.e. during

the inflaton oscillations and decays into Susy SM (or Beyond SM) particles. $\rho_{\tilde{G}}(t_{Re})/\rho_R(t_{Re})$ is estimated as

$$\frac{\rho_{\tilde{G}}(t_{Rh})}{\rho_R(t_{Rh})} \simeq \frac{8\pi}{3} \left(\frac{\rho_{\tilde{G}}(t_e)}{M_{Pl}^2 H^2(t_e)} \right) \quad (30)$$

The inflaton mass is the characteristic scale for the Hubble constant calculated in t_e : $H^2(t_e) \sim m_\phi^2$ and $\rho(t_e) \sim m_\phi^2 M_{Pl}^2$. and this implies the following relations for gravitino abundance

$$\Omega_{\tilde{G}} h^2 \sim 10^{17} \left(\frac{T_{Rh}}{10^9 \text{ GeV}} \right) \left(\frac{\rho_{\tilde{G}}(t_e)}{\rho_c(t_e)} \right) \quad (31)$$

where $\rho_c(t_e) = 3H(t_e)^2 M_{Pl}^2/8\pi$ is the critical energy density during t_e . Eq.(31) can be conveniently rewritten as

$$\Omega_{\tilde{G}} h^2 \simeq \Omega_R h^2 \left(\frac{T_{Rh}}{T_0} \right) \frac{8\pi}{3} \left(\frac{\langle m_{\tilde{G}} \rangle}{M_{Pl}} \right) \frac{n_{\tilde{G}}(t_e)}{M_{Pl} H^2(t_e)} \quad (32)$$

As explicitly shown in Eq.(32), the gravitino mass is up to the inflaton mass and the reheating temperature. However, the inflaton mass is constrained to be $m_\phi \simeq 10^{13}$ GeV or so. On the other hand $T_{Rh}/T_0 \simeq 4.2 \times 10^{14}$ for a successful reheating. As a consequence, a correct abundance of cold dark matter can be recovered for a gravitino mass of $\langle m_{\tilde{G}} \rangle \simeq (10^{-2} \div 1) \times m_\phi \simeq 10^{11} \div 10^{13}$ GeV, constraining W_0 in Eq.(10). As a result the SUSY symmetry breaking scale is expected to be around the gravitino mass. In particular, all other superparticles are assumed to be heavier than the gravitino.

IV. COMMENTS ON STRING NON-PERTURBATIVE CONTRIBUTIONS

Our model could be UV completed in context of string theory. It is commonly retained that in the limit of $\alpha' = l_s^2 \rightarrow 0$, superstrings reduce to supergravity models. However non-perturbative stringy corrections can generate new effective superpotentials which are not allowed at perturbative level. In our framework, stringy corrections can destabilize the gravitino leading to possible phenomenological implications for indirect detection of dark matter. In particular, the initial $U(1)_R$ gauge symmetry can be broken by exotic stringy instantons, i.e. by Euclidean D-brane instantons of open superstring theories or worldsheet instantons in heterotic superstring theory (See [50] for a review on this subject). For example, the generation of $\mu H L$ superpotentials by $E2$ -branes in intersecting D6-brane models was discussed in Ref. [32]. The associated effective lagrangian is

$$\mathcal{L}_{E2} = C^{(1)} \beta^{(1)} H_{u_A} \tau_A^{(1)} + C_i'^{(1)} \gamma^{(1)} L_A^i \tau_A^{(1)} \quad (33)$$

where $\beta^{(1)}, \gamma^{(1)}, \tau^{(1)}$ are fermionic zero modes, which correspond to excitations of open strings attached to $U(1) - E2$, $U(1)' - E2$ and $Sp_L(2) - E2$ respectively. Integrating out fermionic zero modes, one obtains

$$\int d^2\theta \mathcal{W} = \int d^2\theta \int d^2\tau^{(1)} d\beta^{(1)} d\gamma^{(1)} e^{\mathcal{L}_E} \quad (34)$$

$$= M_S e^{-S_{E2}} (C^{(1)} C_i'^{(1)}) H_u L^i$$

where M_S is the string scale and $e^{-S_{E2}}$ is controlled by the geometric scalar moduli which parametrize the 3-cycles, wrapped by the $E2$ -instanton on the Calabi-Yau CY_3 .

On the other hand, in NMSSM scenarios, the introduction of a chiral singlet superfield \mathcal{S}_R can allow the non-perturbative generation of suppressed effective superpotential of the type

$$\mathcal{W}_R = \mu_i \left(\frac{\mathcal{S}_R}{M_S} \right)^n H_u L^i \quad (35)$$

The first term of Eq.(34) is dangerous, since the gravitino has also a coupling with W^\pm, Z, γ, V_R gauge bosons and their related gauginos of the form

$$L_{int} = -\frac{i}{8M_{Pl}} \bar{\psi}_\mu [\gamma^\nu, \gamma^\rho] \gamma^\mu \lambda F_{\nu\rho} \quad (36)$$

As usual, neutral gauginos mix with higgsinos, and their mass eigenstates are neutralinos. So that, from (34) and (36), neutralinos mediate two-body decays $\tilde{G} \rightarrow \gamma\nu, Z\nu, V_R\nu$. In particular $\tilde{G} \rightarrow \gamma\nu$ is the easier decay to constrain since very high energy gamma rays and neutrinos with a peak distribution are produced. The associated decay rate is

$$\Gamma_{\tilde{G} \rightarrow \gamma\nu}^{(0)} = \frac{1}{32\pi} \cos^2 \theta_W \frac{m_\nu}{m_\chi} \frac{m_{\tilde{G}}^3}{M_{Pl}^2} \left(1 - \frac{m_\nu^2}{m_{\tilde{G}}^2} \right)^3 \left(1 + \frac{m_\nu^2}{3m_{\tilde{G}}^2} \right) \quad (37)$$

Now, in our high scale supersymmetry breaking, assuming $m_\chi \simeq 10^{13}$ GeV and $m_{\tilde{G}} \simeq 10^{11}$ GeV, the decay rate is of only $\Gamma^0 \simeq 10^{-20}$ eV corresponding to $\tau^0 \simeq 10^5$ s. This implies that non-perturbative stringy instantons generating the operator (34) can be very dangerous: they completely destabilize gravitino Dark Matter and they have to be suppressed in non-perturbative regime. This is possible if specific non-perturbative RR or NS-NS fluxes are wrapped by the instantonic Euclidean D-brane [51]. Calling $\mathcal{N}_{N.P.}$ the non-perturbative suppression factor, this can screen the the bare decay rate as $\Gamma = \mathcal{N}_{N.P.} \Gamma^0$. A suppression factor $\mathcal{N} \simeq 10^{-11}$ in order to get a gravitino cosmological life-time of at least 1 Gyr or so.

On the contrary, operator like (35) can destabilize the gravitinos with an overall suppression $(\langle \phi_S \rangle / M_S)^n$ assuming that the singlet gets a vacuum expectation value. The corresponding decay time has to be suppressed up to a cosmological time scale $\tau = (M_S / \langle \phi_S \rangle)^n \tau^0 > 1$ Gyr. For $n = 1$, $\langle \phi_S \rangle \simeq 10^{-11} M_S$ saturates the bound. Assuming $M_S = g_S M_{Pl} \simeq 10^{16}$ GeV, the scalar singlet mass is around 100 TeV, which could be reached by the next generation of colliders, with decay channels strongly depending on the completion of our model. On the other hand, for $n > 1$ the scalar singlet field is heavier than 100 TeV. This opens the interesting possibility of super-heavy gravitino decays $\tilde{G} \rightarrow \gamma\nu$ with two

photons and neutrino peaks of energy $E_{CM} \simeq m_{\tilde{G}}/2 \simeq 10^8 \div 10^{13}$ GeV.

The observation of a so high energy neutrinos and photons could be a strong indirect evidence in favor of our scenario. In particular, these very high energy neutrinos can be observed by AUGER, Telescope Array, ANTARES and IceCube. and while eventually they could not be explained by any possible astrophysics sources.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we have discussed some implications of a $R + \zeta R^2$ supergravity model with supersymmetry broken at high scales. As is well known, the Starobinsky (super)gravity is in agreement with Planck data. Then, we showed how this model can also provide a good candidate of Super Heavy Gravitino Dark Matter. Gravitinos can be non-thermally produced during inflationary slow-roll. Intriguingly, in the spaces of parameter of inflaton field and of this gravitino are connected. This model provides a new peculiar prediction: Super-Heavy Gravitinos are produced with a continuous mass spectrum, following the inflaton field. In our framework, CDM data can be constrained by the inflaton potential (and viceversa). Finally, we commented on possible problems in the UV completion of our supergravity model in contest of superstring theories. In particular, even if the gravitino can be protected by R-parity in perturbative supergravity, it will be not protected by any custodial discrete or abelian gauge symmetries in non-perturbative strings regime. The gravitino can be destabilized very fast, even in the limit of $\alpha' \rightarrow 0$. In addition, the famous problem of string moduli stabilization during inflation is still present. But non-perturbative effects can strongly suppress a certain class of operators generated by Euclidean D-branes of worldsheet instantons. It is conceivable that the non-perturbative UV protection cannot avoid all possible R-parity violating gravitino decays. This implies that the gravitino can decay in a cosmological time in several channels. In particular two-body decays $\tilde{G} \rightarrow \gamma\nu$ can produce very high energy peaks of neutrinos and photons, of $E_{CM} \simeq 10^3 \div 10^7$ PeV. The detection of these very high energy neutrinos with a peak-like two-body decay distribution could be a strong indirect hint for our model. On the other hand, we also relate our proposal with the presence of a new scalar singlet at 100 TeV, which could be detected at future high energy colliders.

Our suggestion should extend to a more general class of $f(R)$ -supergravity, like $R + R^n$, with $n > 2$, studied in [15]. Many attempts to unified phantom dark energy and inflation were suggested in contest of $f(R)$ -gravity [53, 54] (see [52] for a review on general aspects of $f(R)$ -gravity). The UV completion of more general $f(R)$ -gravity models can provide a unifying picture of

dark matter, dark energy and inflation^{1 2}. The same mechanism of gravitinos production discussed in Section III could also be implemented in string-inspired climbing scalar pre-inflationary models [59–61]. In this case, a more complicated mass density spectrum of gravitinos is expected and pre-inflationary produced gravitinos should be expected to be part of the CDM composition. However, a detailed analysis deserves a separate analysis beyond the purpose of this letter. Finally, we mention that the parameters space of gravitinos mass can change if a consistent amount of Primordial Black Holes [62–64] were produced during the early Universe. In this case, superheavy gravitinos could have been produced out of thermal equilibrium after the reheating by PBHs evaporation.

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- ¹ The same attitude has inspired recent papers suggesting a unifying picture of dark matter and dark energy from a hidden strong sector [55, 56]. On the other hand, possible implications in direct detection of similar models in the framework of hidden asymmetric standard model were recently discussed in Ref.[57].
- ² We mention that in contest of more generic $f(R)$ -gravity, it remains still unclear the role of primordial black holes, which apparently have a antievaporation instability rendering impossible their evaporation [58].

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